

A CONVENIENT CORRELATION FOR HEAT TRANSFER TO CONSTANT AND VARIABLE PROPERTY FLUIDS IN TURBULENT PIPE FLOW

C. A. SLEICHER and M. W. ROUSE

Department of Chemical Engineering, University of Washington, Seattle, Washington 98195, U.S.A.

(Received 9 April 1973 and in revised form 30 July 1974)

Abstract—A recent equation for calculating heat-transfer coefficients to constant property fluids in pipes is modified to correlate variable property data. The correlating equation is of convenient form, applies to both liquids and gases, correlates constant property results within 10 per cent for $0.1 < Pr < 10^4$ with $10^4 < Re < 10^6$, and correlates variable property data within 20 per cent for $0.7 < Pr < 75$ with $10\,000 < Re < 506\,000$. The equation is compared with the Sieder-Tate equation for liquids and the two equations of Petukhov for gases and liquids. Equations for heat transfer to variable property liquid metals are suggested.

NOMENCLATURE

- A , area of heat exchanger;
- C_p , heat capacity at constant pressure;
- D , inside pipe diameter;
- f , friction factor defined by equation (5);
- h , local heat-transfer coefficient, $q/(T_w - T_b)$;
- k , thermal conductivity;
- Nu , local Nusselt number, hD/k ;
- Pr , local Prandtl number, $C_p\mu/k$;
- q , local heat flux;
- Q , rate of heat transfer in an exchanger;
- Re , local Reynolds number, $DV\rho/\mu$;
- T , absolute temperature;
- U , overall heat-transfer coefficient in an exchanger;
- V , local bulk average velocity in a pipe;
- x , distance from thermal entrance.

where the coefficient 0.023 is recommended by McAdams [1] in place of 0.0243 originally given by Dittus and Boelter, and the Colburn equation

$$Nu = 0.023 Re^{0.8} Pr^{1/3} \quad (2)$$

These equations are intended to apply only to fully-developed coefficients (absence of hydrodynamic and thermal entry effects) and to constant property fluids (low heating rates).

When heating or cooling rates are high, the variation of fluid properties, principally viscosity, with temperature causes data to depart from uniform property data. Several ways of accounting for this departure have been proposed. For liquids the most widely used of these was suggested by Sieder and Tate [2] and amounts to applying a viscosity correction to equation (2):

$$Nu_b = 0.023 Re_b^{0.8} Pr_b^{1/3} (\mu_b/\mu_w)^{0.14} \quad (3)$$

where the subscripts b and w indicate that the relevant physical properties are evaluated at the bulk and wall temperatures respectively.

Equation (3) does not apply well to gases, though it is sometimes used for this purpose. There is an extensive literature on heat transfer to gases with large wall-to-bulk temperature differences, and a variety of correlation methods have been proposed. Good reviews of the subject have been written by Petukhov [3] and Kays and Perkins [4].

It is worth noting that most of the heat-transfer coefficients upon which equations (1) and (2) were based and all of the coefficients upon which equation (3) was based are coefficients that are averages over the length of an exchanger. In these cases only the inlet and outlet bulk temperatures were measured and used to compute the coefficients from

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (1) \qquad Q = UA(\Delta T)_m \quad (4)$$

Greek symbols

- μ , viscosity;
- ρ , density.

Subscripts

- 0, constant property case, equation (5) or (9);
 - b , evaluated at local bulk temperature;
 - w , evaluated at local wall temperature;
 - f , evaluated at local film temperature
- $T_f = (T_b + T_w)/2$.

INTRODUCTION

FOR MANY years the two most widely used correlations for computing heat transfer coefficients in pipes have been the Dittus-Boelter equation, usually given in the form

where $(\Delta T)_{lm}$ is the logarithmic mean of the temperature difference (between the hot and cold streams) at one end of the exchanger and at the other end of the exchanger. No error is introduced by this procedure when the exchanger is long (so that entrance effects are negligible) and when physical properties are effectively uniform everywhere, for in this case the coefficients are uniform and equation (4) follows exactly. On the other hand, when physical properties vary in the exchanger, the heat-transfer process is not described by equation (4) and consequently the average coefficients calculated "by definition" from equation (4) are difficult to correlate. Point (or local) coefficients should be easier to correlate because they are more sensitive to local conditions than to upstream conditions and because physical property variations over the pipe radius are generally much less than over the exchanger length. For these reasons, among others, there is considerable scatter of data in the literature. Only local coefficients will be considered in this paper.

It is known that the Dittus-Boelter equation is inaccurate over certain ranges of the Reynolds and Prandtl numbers. It is apparently 10–25 per cent high for gases and as much as 40 per cent low at intermediate Prandtl numbers and high Reynolds number. Because of these and other uncertainties when the equation is used for design, the designer usually incorporates a liberal safety factor. Although this conservative practice is sound, it is possible that safety factors and costs could be reduced somewhat if equations more reliable than (1), (2), and (3) were available. These considerations led Petukhov to engage in a long study of heat-transfer coefficients in pipes. This work as well as others is presented in his admirably thorough review [3].

In his paper Petukhov presents two correlations for *constant property*, fully-developed heat-transfer coefficients in pipes. One of them correlates his calculations within 2 per cent but is quite complex. Since neither the mathematical model nor heat-transfer data are likely to be accurate within 2 per cent, and since it is seldom if ever that coefficients of that accuracy are required, Petukhov presents a simpler equation which correlates his results within 10 per cent for $0.5 < Pr < 2000$ and $10^4 < Re < 5 \times 10^6$. The equation is:

$$Nu_0 = \frac{RePr(f/8)}{1.07 + 12.7(Pr^{2/3} - 1)\sqrt{f/8}} \quad 0.5 < Pr < 2000 \quad (5)$$

where

$$f = (1.82 \log Re - 1.64)^{-2} \quad 10^4 < Re < 5 \times 10^6.$$

This equation has been critically evaluated by Webb [5], who concludes that it provides a much better correlation of heat- and mass-transfer data than do the Dittus-Boelter or Colburn equations.

Petukhov also presents the results of a theoretical analysis of heat transfer to variable property fluids. The analysis incorporates assumptions about the effect of physical property variation on eddy diffusivity, and the results are given in the form of inconvenient

integrals. Consequently, Petukhov recommends a Sieder-Tate type correction to equation (5) for variable property liquids:

$$Nu_b = Nu_0(\mu_b/\mu_w)^n \quad (6)$$

where $n = 0.11$ for heating and 0.25 for cooling. This equation has been critically evaluated by Hufschmidt, Burck and Riebold [6], who in a well-designed experiment took much data on heating of water at large temperature differences. They found that their data were well correlated by a modified form of equation (6):

$$Nu_b = Nu_0(Pr_b/Pr_w)^{0.11}. \quad (7)$$

The difference between equations (6) and (7) is usually small because for most liquids the ratio C_p/k is only a weak function of temperature.

For variable property *gases* Petukhov suggests

$$Nu_b = Nu_0(T_w/T_b)^n \quad (8)$$

$$n = -a \log_{10}(T_w/T_b) - 0.36$$

where $a = 0$ for cooling and 0.3 for heating.

Recently Notter and Sleicher [7] also published an analysis of heat transfer to constant property fluids in pipes. Their analysis was completed before the appearance of Petukhov's paper, though they overlooked an earlier paper by Petukhov and Kirillov [8] in which equation (5) was first given. Notter and Sleicher's calculations were correlated within 10 per cent by

$$Nu_0 = 5 + 0.015 Re^a Pr^b \quad \begin{array}{l} 0.1 < Pr < 10^4 \\ 10^4 < Re < 10^6 \end{array} \quad (9)$$

$$a = 0.88 - 0.24/(4 + Pr)$$

$$b = 1/3 + 0.5e^{-0.6Pr}$$

In reference [7] the coefficient in equation (9) is given as 0.016 rather than 0.015. As shown later in this paper, however, the value 0.015 correlates both constant and variable property data a little better.

For gases ($0.6 < Pr < 0.9$) equation (9) simplifies within 4 per cent to

$$Nu_0 = 5 + 0.012 Re^{0.83} (Pr + 0.29). \quad (9a)$$

A comparison of equations (5) and (9) is revealing. Despite their entirely different structure, the correlations give surprisingly similar results. Over the major portion of parameter space from $0.1 < Pr < 10^4$ and $10^4 < Re < 5 \times 10^6$ these equations differ by less than 5 per cent, and the maximum difference is 9 per cent, which occurs near $Re = 10^6$, $Pr = 1.4$. Consequently, the choice between the two equations is largely a matter of taste. The equation of Petukhov and Kirillov is appealingly elegant, whereas that of Notter and Sleicher retains some of the simplicity of the Dittus-Boelter equation and reveals more clearly the way in which Nu varies with Re and Pr . We have made many calculations with both equations in the preparation of this paper and find that in practice equation (9) is easier to use; it requires fewer entries on a desk calculator than does equation (5).

DEVELOPMENT OF VARIABLE PROPERTY CORRELATION

The apparent success of equation (9) in correlating constant property data over a wide range of Prandtl and Reynolds numbers led us to investigate whether it could be modified to account for property variation. The equation was modified by applying to it a Sieder-Tate type correction factor and by determining the best temperature at which to evaluate the Nusselt, Prandtl, and Reynolds numbers. The modified equation is:

$$\begin{aligned} Nu_i &= 5 + 0.015 Re_j^a Pr_k^b (\mu_i/\mu_w)^n \\ a &= 0.88 - 0.24/(4 + Pr_k) \\ b &= 1/3 + 0.5e^{-0.6Pr_k} \end{aligned} \quad (10)$$

where the subscripts denote some reference temperature at which the parameter is evaluated. The question to be answered is: What are the best values of i, j, k, l and n ?

Consideration was limited to three choices of reference temperature—the local wall temperature T_w , the local bulk temperature T_b , and the local “film” temperature $T_f \equiv (T_w + T_b)/2$. Hence i, j, k and l could each be either w, f , or b . Values of n studied were 0, 0.05, 0.08, 0.1, 0.12, 0.14, 0.16, 0.18, 0.20, 0.25, 0.30 and 0.40.

The values of i, j, k, l and n were determined by fitting equation (10) to selected data in the literature. These data were chosen to meet the following conditions: absence of thermal and hydrodynamic entry effects, wide range of fluid properties over the pipe radius, local rather than mean values of heat-transfer coefficient, general care with which the experiment was designed and performed, and completeness of the description of equipment and results. A computer was programmed to calculate a set of Nu from equation (10) for comparison to the experimental values. Note that for each experimental value of h from the literature there were three experimental values of Nu ($i = b, f$, and w), and each was compared to 216 values of Nu calculated from equation (10), there being three values of Re , three of Pr , twelve of n , and two of l (b and f).

An examination of the computer output revealed that one correlation was slightly superior to all others and far superior to most. Surprisingly and fortunately, the best value of n is zero with $i = b, j = f$, and $k = w$. Thus the final correlation is

$$\begin{aligned} Nu_b &= 5 + 0.015 Re_f^a Pr_w^b & 0.1 < Pr < 10^5 \\ & & 10^4 < Re < 10^6 \\ a &= 0.88 - 0.24/(4 + Pr_w) \\ b &= 1/3 + 0.5e^{-0.6Pr_w} \end{aligned} \quad (11)$$

For gases ($0.6 < Pr < 0.9$) the equation becomes

$$Nu_b = 5 + 0.012 Re_f^{0.83} (Pr_w + 0.29). \quad (11a)$$

COMPARISON OF EQUATIONS TO DATA

This correlation and Petukhov's equation (8) will now be compared to data on gases, following which data on liquids will be compared to the Sieder-Tate equation (3), Petukhov's equation (6), and equation (11).

Figure 1 compares calculations to four studies on gases. The data shown are all for fully-developed turbulent flow in the absence of entrance effects but cover a wide range of Reynolds number, wall-to-bulk temperature difference, and distance from thermal entrance. Some other relevant studies from the literature could not be exploited because sufficiently complete data were not given. The points of Kays and Leung [9] are effectively constant property results and were obtained on air. The original data of Deissler and Eian [10], kindly supplied to us by Deissler, consisted of seventy-seven runs on air taken at $x/D = 93$ in which $T_w - T_b$ ranged from 71 to 905°F with $9400 < Re_b < 506\,000$. To reduce the number of points from this study, we choose only even numbered runs and eliminated two runs in which the bulk Reynolds number was less than 10 000. The remaining thirty-seven data points are shown. Leitchuk and Dyadyakin [11] report detailed measurements on air with $T_w - T_b$ up to 1070°F and x/D ranging from 1.1 to 133. Shown on the figure are their thirty points at $x/D = 100$. Perkins and Worsoe-Schmidt [12] used precooled nitrogen to achieve temperature differences as high as 1934°F and T_w/T_b up to 7.54. Shown on Fig. 1 are their data (twenty-eight points) at $x/D = 40, 65, 89$ and 119, except for runs in which laminar-turbulent transition effects or condensation was noted.

The Sieder-Tate equation (3) is not shown on Fig. 1, since the data deviate from it by as much as a factor of 2. Evidently both equations (11) and (8) correlate the wide range of data tolerably well, though deviations of 20–25 per cent occur with equation (11) when T_w/T_b exceeds 2. The average deviation of the 120 points shown is 6.8 per cent for equation (8) and 6.9 per cent for equation (11) or (11a). It is evident, however, that the points scatter about equally above and below equation (11) whereas most of the calculations with equation (8) lie below the data. This is a consequence of over-correction by the T_w/T_b term in equation (8) at T_w/T_b less than about 2. Petukhov [3] states that equation (8) correlates his analytical results, but different values of the constants fit the data on Fig. 1 better. We find that a better fit to heating data occurs when equation (8) is modified to

$$Nu_b = Nu_0 (T_w/T_b)^n \quad (8a)$$

with

$$n = -\log_{10}(T_w/T_b)^{1/4} + 0.3$$

or equation (9a) to

$$\begin{aligned} Nu_b &= 5 + 0.012 Re_b^{0.83} & 0.6 < Pr < 0.9 \\ &\times (Pr_b + 0.29)(T_w/T_b)^n & 10^4 < Re < 10^6 \\ n &= -\log_{10}(T_w/T_b)^{1/4} + 0.3 & 1 < T_w/T_b < 5 \\ & & x/D > 40. \end{aligned} \quad (12)$$

This equation is also shown on Fig. 1. The average deviation of the data is 4.2 per cent and the maximum is 18 per cent with only 4 of the 120 points exceeding 10 per cent. Equations (11) or (11a), (8a), and (12) give very similar results for $T_w/T_b < 2$, and as T_w/T_b increases above about 2, equations (8a) and (12) correlate the

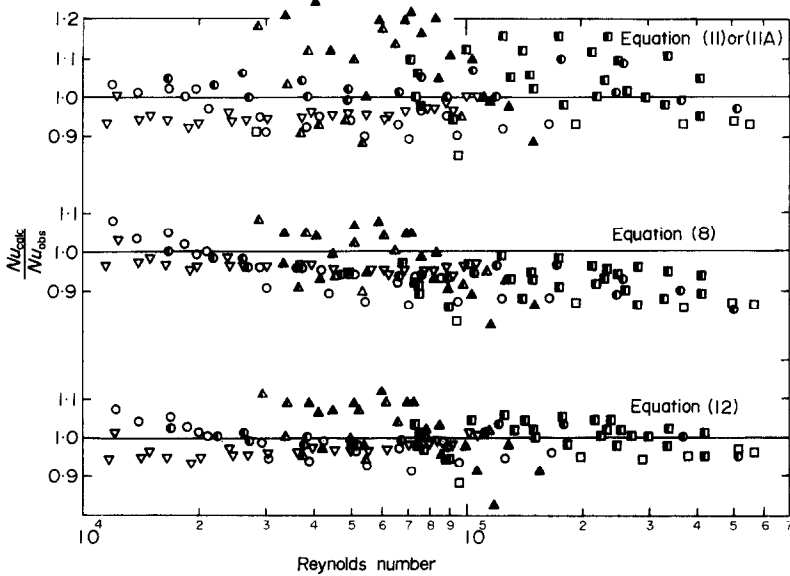


FIG. 1. Comparison of data on gases to three equations.

	T_w/T_b		
	0-1.4	1.4-2	> 2
Deissler and Eian [10]	○	●	
Kays and Leung [9]	▽		
Lel'chuk and Dyadyakin [11]	□	■	
Perkins and Worsoe-Schmidt [12]		▲	▲

data better. Equation (11a) is, of course, much the simplest. Note that its use requires knowledge of three gas properties (k , μ and Pr) each at a different temperature whereas in equation (12) the same properties are evaluated at one temperature.

A comparison of gas data to Petukhov's equation (5) was made with Nu evaluated at T_b , Re at T_f , and Pr at T_w , but results were less satisfactory than equations (11) or (12). We also compared the data on Fig. 1 to the four correlating equations given on pp. 7-164 of Kays and Perkins [4]. The best of these we judge to be the "wall temperature correlation" of Perkins and Worsoe-Schmidt [12], which has the same average deviation from the data as equation (11) but is slightly better than equation (11) for the variable property data and inferior at low values of T_w/T_b . For the 120 data points used here, equation (12) gives a lower average deviation and a lower maximum deviation than any equation we have found.

Equations (3), (6) and (11) will now be compared to data for fluids at high Prandtl (or Schmidt) number—first for the constant property cases and then for variable properties.

The data of Allen and Eckert [13] on water and of Malina and Sparrow [14] are particularly well adapted for comparison to constant property correlations because of the systematic way in which the experiments were designed: At each of four constant values of the bulk Prandtl number (3, 8, 48 and 75), the bulk Reynolds number was held constant (at one of several

values) while the wall-to-bulk temperature difference was systematically varied. Thus extrapolation to the constant property case, $T_w - T_b = 0$, is facilitated. There is a total of eighteen $Pr-Re$ combinations covering the following ranges: $Pr_b = 3$, $14\,500 < Re_b < 101\,000$; $Pr_b = 8$, $13\,000 < Re_b < 111\,000$; $Pr_b = 48$, $14\,100 < Re_b < 43\,100$; $Pr_b = 75$, $12\,000 < Re_b < 27\,500$. The data extrapolated to $T_w - T_b = 0$ for the eighteen $Re-Pr$ combinations are shown as the triangles on Fig. 2. Also shown are the mass-transfer data of Harriot and Hamilton [15] at Schmidt numbers (mass-transfer equivalent of Prandtl number) of 930 and 9810 and three points calculated by Notter and Sleicher [16] at high Schmidt number and for Reynolds numbers beyond the range of Harriot and Hamilton's data.

It is clear from Fig. 2 that equations (5) and (9) correlate the data much better than the Dittus-Boelter equation (1), in agreement with Webb [5]. The average deviations of the thirty-one points shown are 14 per cent for equation (1), 3.4 per cent for equation (5), and 4.1 per cent for equation (9). Apparently, equations (5) and (9) are both excellent fits to the data over a wide range of Prandtl and Reynolds numbers.

Figure 3 compares some variable property data to the Sieder-Tate equation (3), Petukhov's equation (6), and equation (11). The data shown are those of Allen and Eckert [13] and Malina and Sparrow [14]. Although these studies included data at many values of $T_w - T_b$, it is superfluous to compare all of these to the correlations. Rather we have compared the data only at the

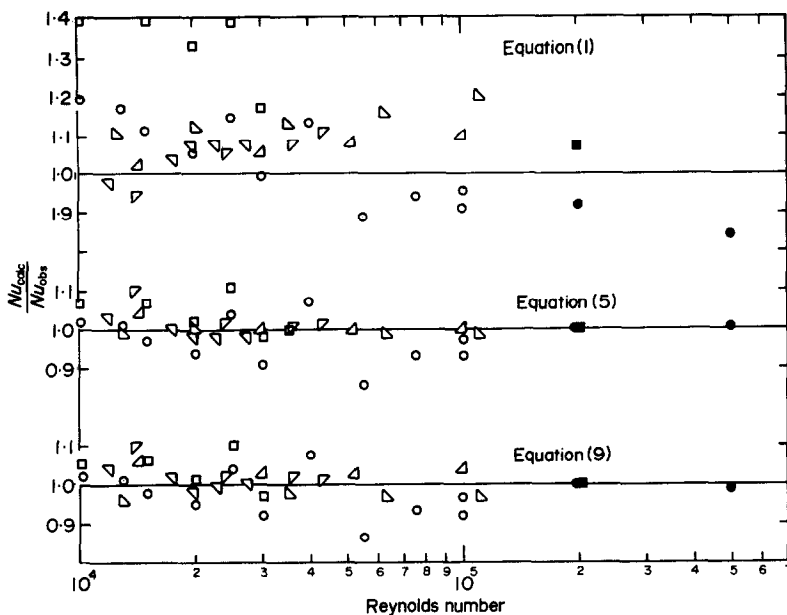


FIG. 2. Comparison of constant property data on liquids to three equations. Allen and Eckert [13], Δ , $Pr = 8$; Malina and Sparrow [14], \triangle , $Pr = 3$, ∇ , $Pr = 48$, ∇ , $Pr = 75$; Harriot and Hamilton [15], \circ , $Sc = 930$, \square , $Sc = 9810$; Notter and Sleicher [16], \bullet , $Sc = 1000$, \blacksquare , $Sc = 10000$.

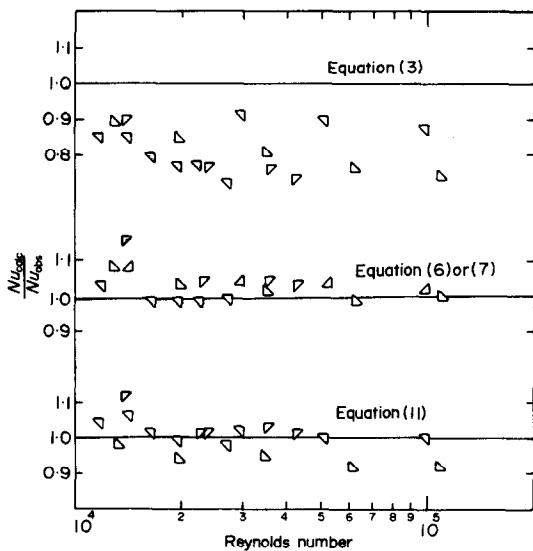


FIG. 3. Comparison of variable property data on liquids to three equations. Allen and Eckert [13], \triangle , $Pr = 3$, $\mu_b/\mu_w = 1.46-2.81$; Malina and Sparrow [14], \triangle , $Pr = 8$, $\mu_b/\mu_w = 1.435$; ∇ , $Pr = 48$, $\mu_b/\mu_w = 1.835-2$; ∇ , $Pr = 75$, $\mu_b/\mu_w = 1.85-2.88$.

approximate *maximum* experimental values of $T_w - T_b$ at each of the eighteen $Re-Pr$ combinations. For these data the values of μ_b/μ_w range from 1.43 to 2.88. The average deviations of the data are 23 per cent for equation (3), 4.1 per cent for equation (6) and 3.9 per cent for equation (11). It is clear, at least for water and the oil used by Malina and Sparrow (a chlorinated biphenyl), that equations (6) and (11) correlate the data about equally well.

The data of Allen and Eckert and Malina and Sparrow cannot provide a test of the relative merits of equations (6) and (7) because the predictions of the two equations differ by less than 0.5 per cent over the range of variables studied.

The foregoing comparisons show that for most design purposes either equation (11) for all fluids or equation (6) for liquids and equation (12) for gases is satisfactory. Equation (11), however, is more convenient to use. For gases equation (11a) is particularly simple and correlates data well to $T_w/T_b < 2$.

One advantage of equation (11) is that it displays clearly the way in which Nu varies with velocity or Reynolds number. It is therefore useful in determining the best way to make a "Wilson plot" for determining the outside heat-transfer coefficient of an exchanger by varying the inside fluid flow rate. The equation shows that $1/U$ should be plotted vs V^{-a} or Re_f^{-a} for extrapolation of V^{-a} or Re_f^{-a} to zero. (At high Reynolds number the 5 in equation (11) is negligible.)

Equation (11) has not yet been adequately tested against cooling data. It is our judgment, however, that equation (11) is the best form of equation (9) for cooling as well as heating. A comparison of equations (11) with equations (6) and (8) for local cooling coefficients would be interesting.

ASYMPTOTIC FORMS

The asymptotic form of equation (11) for high Prandtl number is

$$Nu_b = 0.015 Re_f^{0.88} Pr_w^{1/3} \tag{13}$$

which closely approximates (11) for $Pr > 50$. This

equation in its constant property form has previously been given by Notter and Sleicher [16] as a correlation for high Prandtl number fluids. The equation correlates very well with the data of Malina and Sparrow [14] at $Pr = 48$ and $Pr = 75$ as well as the mass-transfer data of Harriot and Hamilton [15] for Schmidt numbers as high as 100000.

For very low Prandtl number fluids, the liquid metals, the asymptotic form of equation (11) is

$$Nu_b = 5 + 0.015 Re_f^{0.82} Pr_w^{0.83} \quad Pr < 0.04. \quad (14)$$

This equation gives surprisingly close agreement, ± 10 per cent, with the data of Sleicher, Awad and Notter [17] for heat transfer to liquid metals in a pipe at *uniform wall temperature*. Nevertheless, the equation is not as good in the liquid metal range as the equations given in [17]. If we assume that the modification of constant property equation (9) to achieve variable property correlation (11) can also be applied to the liquid metal correlations of reference [17], those correlations become

Uniform wall temp.

$$Nu_b = 4.8 + 0.0156 Re_f^{0.85} Pr_w^{0.93}, \quad Pr < 0.1 \quad (15)$$

Uniform wall flux:

$$Nu_b = 6.3 + 0.0167 Re_f^{0.85} Pr_w^{0.93}, \quad Pr < 0.1. \quad (16)$$

These two correlations are offered tentatively. Although they are good correlations for constant properties, they have not been tested for variable properties.

SUMMARY AND CONCLUSIONS

A single equation, equation (11), correlates local heat-transfer coefficients in pipes for both liquids and gases whose physical properties are temperature dependent. The equation is more convenient to use than Petukhov's separate equations for liquids and gases, and it equals the accuracy of those equations except for gases in which T_w/T_b exceeds 2, in which case equation (12) is recommended. Equation (11) has not been tested against local cooling coefficients, though its deviations from the constant property correlation are in the correct direction for both cooling and heating.

Based on the success of the structure of equation (11), two equations, (14) and (15), are offered tentatively as correlations for heat transfer to variable property liquid metals in pipe flow.

Acknowledgement—We are grateful to R. G. Deissler for kindly supplying us with his original data on air.

REFERENCES

- W. H. McAdams, *Heat Transmission*, 3rd edn. McGraw-Hill, New York (1954).
- E. N. Sieder and G. E. Tate, Heat transfer and pressure drop of liquids in tubes, *Ind. Engng Chem.* **28**, 1429 (1936).
- B. S. Petukhov, Heat transfer and friction in turbulent pipe flow with variable physical properties, *Adv. Heat Transf.* **6**, 503–504 (1970).
- W. M. Kays and H. C. Perkins, in *Handbook of Heat Transfer*, edited by W. M. Rohsenow and J. P. Hartnett, Section 7, McGraw-Hill, New York (1973).
- R. L. Webb, A critical evaluation of analytical solutions and Reynolds analogy equations for turbulent heat and mass transfer in smooth tubes, *Wärme- u. Stoffübertragung* **4**, 197–204 (1971).
- W. Hufschmidt, E. Burck and W. Riebold, Die Bestimmung Örtlicher und Mittlerer Wärmeübergangszahlen in Röhren bei Hohen Wärmestromdichten, *Int. J. Heat Mass Transfer* **9**, 539–565 (1966).
- R. H. Notter and C. A. Sleicher, A solution to the turbulent Graetz problem—III. Fully developed and entry region heat transfer rates, *Chem. Engng Sci.* **27**, 2073–2093 (1972).
- B. S. Petukhov and V. V. Kirillov, The problem of heat exchange in the turbulent flow of liquids in tubes, *Teplotengetika* No. 4, 63–68 (1958).
- W. M. Kays and E. Y. Leung, Heat transfer in annular passages—hydrodynamically developed turbulent flow with arbitrarily prescribed heat flux, *Int. J. Heat Mass Transfer* **6**, 537–557 (1963).
- R. G. Deissler and C. S. Eian, Analytical and experimental investigation of fully developed turbulent flow of air in a smooth tube with heat transfer with variable fluid properties, NACA TN 2629 (1952).
- V. L. Lel'chuk and B. V. Dyadyakin, Heat transfer from a wall to a turbulent current of air inside a tube and the hydraulic resistance at large temperature differentials, *Problems in Heat Transfer*, edited by M. A. Mikheev, AEC-tr-4511 (1962). (Translation of *Voprosy Teploobmena*, Akademii Nauk S.S.S.R., Moskva (1959).)
- H. C. Perkins and P. Worsoe-Schmidt, Turbulent heat and momentum transfer for gases in a circular tube at wall to bulk temperature ratios to seven, U.S.A.E.C. TR SU 247-7 (1964). See also *Int. J. Heat Mass Transfer* **8**, 1011 (1965).
- R. W. Allen and E. R. G. Eckert, Friction and heat-transfer measurements to turbulent pipe flow of water ($Pr = 7$ and 8) at uniform wall heat flux, *J. Heat Transfer* **86**, 301–310 (1964).
- J. A. Malina and E. M. Sparrow, Variable-property, constant-property, and entrance region heat-transfer results for turbulent flow of water and oil in a circular tube, *Chem. Engng Sci.* **19**, 953–962 (1964).
- P. Harriot and R. M. Hamilton, Solid-liquid mass transfer in turbulent pipe flow, *Chem. Engng Sci.* **20**, 1073–1078 (1965).
- R. H. Notter and C. A. Sleicher, The eddy diffusivity in the turbulent boundary layer near a wall, *Chem. Engng Sci.* **26**, 161–171 (1971).
- C. A. Sleicher, A. S. Awad and R. H. Notter, Temperature and eddy diffusivity profiles in NaK, *Int. J. Heat Mass Transfer* **16**, 1565–1576 (1973).
- D. R. Dickinson, Tests of the effect of scale build-up on heat transfer using untreated water under conditions typical of the ETS-1 exhaust duct, AEC R & D Report, BNWL-632 (March 1968).

NOTE ADDED IN PROOF

The difference in Nusselt numbers predicted by equations (6), (7), and (11) are small when applied to the data of Allen and Eckert [13], Malina and Sparrow [14]. We have just become aware, however, of a study by Dickinson [18] which contains data on heat transfer to water at very high heat fluxes. The tests appear to have been carefully performed, local coefficients can be calculated from the data at seven locations, the wall heat flux was uniform, and there was a hydrodynamic entrance section with $L/D = 64$. The observed and calculated results are summarized in the following table. Points near the ends of the test section are not included. For these data the average deviation of equation (6) is 22 per cent and the average deviation of equation (11) is 7 per cent.

L/D	T_b (°F)	T_w (°F)	Re_b	Pr_b	Nu_{obs}	Nu_{obs}/Nu_{calc}	
						Equations (6) or (7)	Equation (11)
36	108	308	323 000	4.20	1160	1.23	1.04
48	115	326	347 000	3.85	1090	1.33	1.13
60	123	333	373 000	3.6	1090	1.35	1.17
72	130	320	397 000	3.3	1190	1.24	1.08
36	112	344	336 000	4.0	1230	1.18	1.01
48	121	351	365 000	3.65	1220	1.20	1.04
60	130	361	397 000	3.3	1205	1.24	1.08
72	139	362	428 000	3.05	1230	1.22	1.08
36	125	272	37 600	3.5	182	1.15	1.03
48	134	275	40 800	3.2	188	1.13	1.02
60	143	288	44 100	2.9	181	1.20	1.09
72	152	289	47 700	2.7	189	1.16	1.07